Definitions about Relations

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5.1

(p. 197) A binary ***relation*** R from X to Y is a subset of $X \times Y$. X and Y are the ***domains*** of R.

5.3

(p. 202) The ***image*** of an element $a \in X$ under a relation R from X to Y is defined as

$$\mathcal{I}_R(a) = \{b \in Y \mid (a, b) \in R\}$$

(p. 202) The ***image*** of a set $A \subseteq X$ under a relation R from X to Y is defined as

$$\mathcal{I}_R(A) = \{b \in Y \mid \exists\, a \in A \mid (a, b) \in R\}$$

(p. 202) The ***inverse*** of a relation R from X to Y is the relation

$$R^{-1} = \{(b, a) \in Y \times X \mid (a, b) \in R\}$$

(p. 203) The ***composition*** of a relation R from X to Y and a relation S from Y to Z is the relation

$$S \circ R = \{(a, c) \in X \times Z \mid \exists\, b \in Y \mid (a, b) \in R \wedge (b, c) \in S\}$$

(p. 205, ex. 5.3.9) The ***identity relation*** on a set X is defined as

$$i_x = \{(x, x) \mid x \in X\}$$

5.4

(p. 205) A relation R on a set X is ***reflexive*** if every element is related to itself:

$$\forall\, x \in X, (x, x) \in R$$

(p. 206) A relation R on a set X is ***symmetric*** if for every pair in the relation, the inverse of the pair is also in the relation:

$$\forall\, x, y \in X,\ \text{if } (x, y) \in R \text{ then } (y, x) \in R$$

(p. 207) A relation R on a set X is ***transitive*** if any time one element is related to a second and the second is related to a third, then the first is also related to the third:

$$\forall\, x, y, z \in X,\ \text{if } (x, y) \in R \wedge (y, z) \in R \text{ then } (x, z) \in R$$