Chapter 7: Definitions about Functions

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7.1 Definition

A *function* from a set X to a set Y is a relation from X to Y such that each $x \in X$ is related to exactly one $y \in Y$:

$$\begin{array}{rll} \forall x \in X, & \exists y \in Y \mid (x, y) \in f & \text{(existence of y)} \\ \wedge & \forall y_1, y_2 \in Y,\ ((x, y_1) \in f \wedge (x, y_2) \in f) \rightarrow y_1 = y_2 & \text{(uniqueness of y)} \end{array}$$

We denote that value y as $f(x)$. We call X the *domain* of f and Y the *codomain*.
We write $f : X \rightarrow Y$ to mean "f is a function from X to Y."

7.2 Function equality

For $f, g : X \rightarrow Y$, $f = g$ if $\forall x \in X, f(x) = g(x)$

7.4 Images and inverse images

Suppose $f : X \rightarrow Y$ and $A \subseteq X$.

The *image* of A under f is $F(A) = \{y \in Y \mid \exists\ x \in A \mid f(x) = y\}$

The *inverse image* of a set $B \subseteq Y$ under a function $f : X \rightarrow Y$ is $F^{-1}(B) = \{x \in X \mid f(x) \in B\}$