Chapter 5 Assessment

1. Choose one of the following exercises:

   (a) Find the volume under the surface $z = x^2y$ and above the triangle in the xy-plane with vertices $(1, 0)$, $(2, 1)$, and $(4, 0)$.

   (b) Find the volume bounded by the cylinder $x^2+y^2 = 4$ and the planes $z = 0$ and $y+z = 3$.

   (c) Find the volume above the paraboloid $z = x^2+y^2$ and below the half-cone $z = \sqrt{x^2 + y^2}$.

2. Choose one of the following exercises:

   (a) Consider the lamina that occupies the region D bounded by the parabola $x = 1 - y^2$ and the coordinate axes in the first quadrant with density function $\rho(x, y) = y$. Find the mass and center of mass of the lamina.

   (b) A lamina occupies the part of the disk $x^2 + y^2 \leq a^2$ that lies in the first quadrant. Find the centroid of the lamina. Find the center of mass of the lamina if the density function is $\rho(x, y) = xy^2$.

   (c) Find the centroid of a right circular cone with height h and base radius a. (Place the cone so that its base is in the xy-plane with center at the origin and its axis along the positive z-axis.)

3. Choose one of the following exercises:

   (a) Use the transformation $u = x - y$, $v = x + y$ to evaluate

   $$\iint_R \frac{x - y}{x + y}\, dA$$

   where R is the square with vertices $(0, 2)$, $(1, 1)$, $(2, 2)$, and $(1, 3)$.

   (b) Use the transformation $x = u^2$, $y = v^2$, $z = w^2$ to find the volume of the region bounded by the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$ and the coordinate planes.

   (c) Use the change of variables formula and an appropriate transformation to evaluate $\iint_R xy\, dA$, where R is the square with vertices $(0, 0)$, $(1, 1)$, $(2, 0)$, and $(1, -1)$.