Final Assessment *(Due no later than 8am, Tue Dec 8th.)*

Choose one lettered exercise from each of the 5 sections below. Write them up according to the usual style guidelines (with above and beyond for each), scan them to pdf and submit the pdf to canvas.

1. Choose one of the following exercises:

   (a) Find an equation of the plane that is parallel to the plane $x+5y-z+8=0$ and contains the point $(1,1,4)$.

   (b) True or false? Explain your reasoning.

   i. If $\mathbf{u}\cdot\mathbf{v}=\mathbf{0}$, then $\mathbf{u}=\mathbf{0}$ or $\mathbf{v}=\mathbf{0}$.

   ii. If $\mathbf{u}\times\mathbf{v}=\mathbf{0}$, then $\mathbf{u}=\mathbf{0}$ or $\mathbf{v}=\mathbf{0}$.

   iii. If $\mathbf{u}\cdot\mathbf{v}=\mathbf{0}$ and $\mathbf{u}\times\mathbf{v}=\mathbf{0}$, then $\mathbf{u}=\mathbf{0}$ or $\mathbf{v}=\mathbf{0}$.

   (c) Sketch the surface whose equation in spherical coordinates is $\rho=a(1-\cos(\varphi))$ where $a>0$ is a constant. Which familiar fruit does the surface resemble?

2. Choose one of the following exercises:

   (a) Find $\mathbf{T}(0)$, $\mathbf{N}(0)$, and $\mathbf{B}(0)$ for the curve

   $$\mathbf{r}(t)=\left\langle 2\cos(t),\ 2\cos(t)+\frac{3}{\sqrt{5}}\sin(t),\ \ \cos(t)-\frac{6}{\sqrt{5}}\sin(t)\right\rangle.$$

   (b) At time $t=0$ a particle at the origin of an xyz-coordinate system has a velocity vector of $\mathbf{v}_0=\langle 1,\ 2,\ -1\rangle$. The acceleration function of the particle is $\mathbf{a}(t)=\langle 2t^2,\ 1,\ \cos(2t)\rangle$.

   i. Find the position function of the particle.

   ii. Find the speed of the particle at time $t=1$.

   (c) Find the arc length parameterization, $\ell(s)$, of the line through $P(-1,4,3)$ and $Q(0,2,5)$ that orients in the direction of $\vec{PQ}$ and has $\ell(0)=P$.

3. Choose one of the following exercises:

   (a) Find all points on the surface $z=2-xy$ at which the normal line passes through the origin.

   (b) Locate all relative minima, relative maxima, and saddle points of the function $f(x,y)=x^2+3xy+3y^2-6x+3y$.

   (c) Respond to the questions in a blended narrative (i.e. don't address (i), (ii), and (iii) separately, but rather connect them in a single narrative):

   i. How are the directional derivative and the gradient of a function related?

   ii. Under what conditions is the directional derivative of a differentiable function 0?

   iii. In what direction does the directional derivative of a differentiable function have its maximum value? Its minimum value?

4. Choose one of the following exercises:

(a) Find the centroid of the solid bounded by $y = x^2$, $z = 0$, and $y + z = 4$.

(b) Use the transformation $u = x - 3y$, $v = 3x + y$ to find

$$\iint_R \frac{x - 3y}{(3x + y)^2}\, dA$$

where R is the rectangular region enclosed by the lines $x - 3y = 0$, $x - 3y = 4$, $3x + y = 1$, and $3x + y = 3$.

(c) Use a double integral to find the area of the region enclosed by the rose $r = \cos(3\theta)$.

5. Choose one of the following exercises:

(a) Let the surface, S, be the portion of the paraboloid $z = 1 - x^2 - y^2$ for which $z \geq 0$, and let C be the circle $x^2 + y^2 = 1$ that forms the boundary of S as it hits the xy-plane. Assuming that S is oriented up, build the integrals and use Sage/cocalc (or similar) to evaluate and verify Stokes' Theorem in the case where

$$\mathbf{F} = \langle x^2 y - z^2,\ y^3 - x,\ 2x + 3z - 1 \rangle.$$

(b) Evaluate $\oint_C y\, dx - x\, dy$ where C is the cardioid $r = a(1 + \cos(\theta))$ with $0 \leq \theta \leq 2\pi$ and $a > 0$ a constant.

(c) Use the Divergence Theorem to find the flux of $\mathbf{F}$ across the surface S with outward orientation where

$$\mathbf{F} = \langle x - z,\ y - x,\ z - y \rangle$$

and S is the surface of the cylindrical solid bounded by $x^2 + y^2 = a^2$, $z = 0$, and $z = 1$.